Nuclear Magnetic Moments of W^{183}, Os^{187}, and Fe^{57}†

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(Received 31 May 1962; revised manuscript received 22 October 1962)

By making use of a single-particle potential which does not have axial symmetry, we account for the anomalously low magnetic moments of W^{183}, Os^{187}, and Fe^{57}. The low-energy spectrum of W^{183} and observed electromagnetic transitions are treated in detail. We find good agreement between the calculations and the experimental observations.

THE nuclei W^{183}, Os^{187},[1] and Fe^{57},[2] are similar in that they have an odd neutron, ground-state spins of $\frac{1}{2}$, and anomalously small magnetic moments when compared with the values calculated with single-particle wave functions obtained from an axially symmetric harmonic oscillator potential. In this paper, we apply the single-particle wave functions of an asymmetric harmonic oscillator potential to a calculation of these magnetic moments. Using this single-particle potential, together with an asymmetric rotor Hamiltonian, we also compute certain quantities associated with the low-lying energy levels in W^{183}. The magnetic moment calculation indicates that the asymmetric oscillator potential affords a better description of the physical situation than the axially symmetric oscillator. The calculations on W^{183} point out a discrepancy between the single-particle potential used in the calculation and the hydrodynamic model's method of computing rotational moments of inertia.[3]

The Hamiltonian which we use in our calculations is

$$H=H_{\rm sp}+H_R, \tag{1}$$

where

$$H_{\rm sp}=\sum_{i=1}^{3}\left(\frac{P_i^2}{2m}+\omega_i^2X_i^2+Cl_i\cdot S_i+Dl_i^2\right) \tag{2}$$

and

$$H_R=\sum_{i=1}^{3}A_i(I_i-J_i)^2. \tag{3}$$

In $H_{\rm sp}$, we take

$$\begin{aligned}\omega_1&=\omega_0(1+\delta/3+\epsilon),\\ \omega_2&=\omega_0(1+\delta/3-\epsilon),\\ \omega_3&=\omega_0(1-2\delta/3),\end{aligned} \tag{4}$$

which differs from the single-particle Hamiltonian used by Nilsson[4] only in the inclusion of ϵ in ω_1 and ω_2. For C and D, we use the numerical values recommended in reference 1, and in the calculation of single-particle wave functions, we ignore the interaction between different shells of our spherical harmonic oscillator basis representation. Here $\mathbf{l}$ and $\mathbf{s}$ are the single-particle orbital and intrinsic angular momenta and

$$\mathbf{l}+\mathbf{s}=\mathbf{J}, \tag{5}$$

$$\mathbf{J}+\mathbf{R}=\mathbf{I}, \tag{6}$$

where $\mathbf{R}$ is the rotor angular momentum and $\mathbf{I}$ is the total angular momentum of the system; Ω is the projection of $\mathbf{J}$ on a body fixed 3-axis and K is the projection of $\mathbf{I}$ on the same axis (for $\epsilon=0$, it is the nuclear symmetry axis). A_1, A_2, and A_3 are the three rotational inertia constants.

Because the single-particle potential is not spherically symmetric, $\mathbf{J}$ is not a constant of the motion, just as in the axially symmetric oscillator. Here we have the additional complication for $\epsilon\neq0$, that Ω is no longer a constant of the motion, but each single-particle state contains many Ω values, which differ from each other by an even integer. Similarly, when $A_1\neq A_2$, K will no longer be a constant of the motion but each $\mathbf{I}$ state may contain K, $K\pm2$, $K\pm4\cdots\pm|I|$. Due to requirements of rotational symmetry, $(K-\Omega)$ must be an even integer and this is accomplished by restricting both of them to the values $+\frac{1}{2}$, $-\frac{3}{2}$, $+\frac{5}{2}$, $-\frac{7}{2}$, $\cdots$. For our Hamiltonian, the good quantum numbers are N, the shell number of the single-particle wave function and $\mathbf{I}$, the spin of the system.

The solution to the Hamiltonian of Eq. (1) is of the form

$$\Psi^I(\lambda)=\left(\frac{2I+1}{16\pi^2}\right)^{1/2}\sum_K{}' C_K{}^I(\lambda)\sum_{J,\Omega}{}' n_\Omega{}^J(D_{M,K}{}^I\chi_\Omega{}^J+(-1)^{I-J}D_{M,-K}{}^I\chi_{-\Omega}{}^J), \tag{7}$$

where the primed summations indicate the restriction on possible values of K and Ω; λ is a label to specify which state of a given $\mathbf{I}$ we are discussing; $\sum_{J,\Omega}{}' n_\Omega{}^J\chi_\Omega{}^J$ is a solution of $H_{\rm sp}$, which we obtained for $N=0$ to 7 on an electronic computer, using the matrix diagonalization subroutine ANF-202. $D_{MK}{}^I$ is a solution of the symmetric rotor problem. The matrix elements for the asymmetric rotor, which are used for computing the coefficients $C_K{}^I(\lambda)$, and the spacings of the rotational

† Based on work performed under the auspices of the U. S. Atomic Energy Commission.

[1] B. R. Mottelson and S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Skrifter 1, No. 8 (1959).

[2] R. D. Lawson and M. H. Macfarlane, Nucl. Phys. 24, 18 (1961).

[3] A. Bohr, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. 26, No. 14 (1952).

[4] S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. 29, No. 16 (1955).

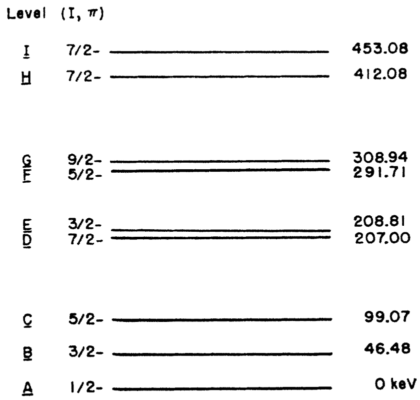

FIG. 1. The energy-level spacings are taken from J. J. Murray, F. Boehm, F. P. Marmier, and J. W. M. DuMond, Phys. Rev. **97**, 1007 (1955). The spin assignments are those of reference 6. Only the level at 453 keV is not considered to arise from the ground-state single-particle wave function.

energy levels are given in a paper by Hecht and Satchler.[5] This useful paper also gives formulas for calculating many other nuclear properties. We reproduce from reference 5 the equations which are relevant to the present calculation.

First we define

$$\langle a+b\rangle = \sum_{J,\Omega}{}' n_\Omega{}^J n_{-(\Omega-1)}{}^J(-1)^{J-\frac{1}{2}} \times[(J+\Omega)(J-\Omega+1)]^{1/2}, \quad (8)$$

$$\langle B\rangle = \sum_{J,\Omega}{}' n_\Omega{}^J n_{-(\Omega+1)}{}^J(-1)^{J-\frac{1}{2}} \times[(J-\Omega)(J+\Omega+1)]^{1/2}, \quad (9)$$

$$\langle \Omega\rangle = \sum_{J,\Omega}{}' (n_\Omega{}^J)^2\Omega, \quad (10)$$

$$\langle K_R\rangle^2 = \sum_{J,\Omega}{}' (n_\Omega{}^J)^2(K-\Omega)^2, \quad (11)$$

$$\langle S^0\rangle = \tfrac{1}{2}\sum_{l,\Omega}{}' (a_{l,\Omega-\frac{1}{2},1/2})^2-(a_{l,\Omega+\frac{1}{2},-\frac{1}{2}})^2, \quad (12)$$

$$\langle S^-\rangle = \sum_{l,\Omega}{}'(-1)^l(a_{l,\Omega-\frac{1}{2},1/2})(a_{l,-\Omega+\frac{1}{2},1/2}), \quad (13)$$

where

$$a_{l,\Omega-\Sigma,\Sigma} = \sum_J \langle l,\tfrac{1}{2},\Omega-\Sigma,\Sigma|J,\Omega\rangle n_\Omega{}^J. \quad (14)$$

The quantity $\langle a+b\rangle$ corresponds to the decoupling parameter for spin $\frac{1}{2}$ bands in the axially symmetric theory; $\langle B\rangle$ corresponds to the Coriolis interaction between two single-particle states in the axially symmetric theory. $\langle K_R\rangle^2$ is a measure of the square of the 3-component of the rotor angular momentum. From Eq. (3), we can see that, for large values of A_3, states which differ markedly in $\langle K_R\rangle^2$ will also be far apart in energy. In the symmetric theory, it is assumed that A_3 is sufficiently large that $\langle K_R\rangle^2=0$ for low-lying rotational levels.

[5] K. T. Hecht and G. R. Satchler, Nucl. Phys. **32**, 286 (1962).

The experimentally observed quantities which we compute are the nuclear magnetic moment and the energy level spacings. The one nucleus whose energy level spacings we shall examine is W^{183}, where many levels have been observed and spin assignments made. The single-particle wave function which we use for W^{183} is such that $\langle K_R\rangle^2$ is about the same for $K=\frac{1}{2}$ and $-\frac{3}{2}$ and considerably larger for other values of K. In computing energy level spacings, we shall take into account only the $K=\frac{1}{2}$ and $-\frac{3}{2}$ projections of I.

For the magnetic moment, μ, we have[5]

$$\mu = g_R\{\tfrac{1}{2}-\tfrac{1}{3}[\langle\Omega\rangle+\langle a+b\rangle]\}+\tfrac{1}{3}g_s[\langle S^0\rangle+\langle S^-\rangle], \quad (15)$$

where $g_R\simeq Z/A=0.4$ nuclear magneton and g_s is taken as -3.826 nuclear magnetons.

We now set

$$A^+=A_1+A_2, \quad (16)$$

$$A^-=A_1-A_2, \quad (17)$$

and

$$\rho=A_3[\langle-\tfrac{3}{2}{}_R\rangle^2-\langle\tfrac{1}{2}{}_R\rangle^2], \quad (18)$$

where the matrix elements in Eq. (18) are defined in Eq. (11).

The energy levels relative to the ground state are[5]

$$E_{(I=3/2)}=A^+(1+\langle a+b\rangle)+A^-\langle B\rangle+\tfrac{1}{2}\rho \pm\tfrac{1}{2}\{[A^+(1+\langle a+b\rangle)+A^-\langle B\rangle-\rho]^2 +3[A^+\langle B\rangle+A^-(1+\langle a+b\rangle)]^2\}^{1/2}, \quad (19)$$

$$E_{(I=5/2)}=A^+(3.5-0.25\langle a+b\rangle)-0.25A^-\langle B\rangle+\tfrac{1}{2}\rho \pm\tfrac{1}{2}\{[A^+(1-1.5\langle a+b\rangle)-1.5A^-\langle B\rangle-\rho]^2 +8[A^+\langle B\rangle-A^-(1.5-\langle a+b\rangle)]^2\}^{1/2}, \quad (20)$$

$$E_{(I=7/2)}=A^+(7+1.5\langle a+b\rangle)+1.5A^-\langle B\rangle+\tfrac{1}{2}\rho \pm\tfrac{1}{2}\{[A^+(1+2\langle a+b\rangle)+2A^-\langle B\rangle-\rho]^2 +15[A^+\langle B\rangle+A^-(2+\langle a+b\rangle)]^2\}^{1/2}, \quad (21)$$

and

$$E_{(I=9/2)}=A^+(11.5-0.75\langle a+b\rangle)-0.75A^-\langle B\rangle+\tfrac{1}{2}\rho \pm\tfrac{1}{2}\{[A^+(1-2.5\langle a+b\rangle)-2.5A^-\langle B\rangle-\rho]^2 +24[A^+\langle B\rangle-A^-(2.5-\langle a+b\rangle)]^2\}^{1/2}. \quad (22)$$

We then use the level scheme given in Fig. 1 and, inserting the experimentally observed level spacings, we compute values for A^+, A^-, ρ, $\langle a+b\rangle$, and $\langle B\rangle$. The analysis which we make here corresponds to that of Kerman[6] except that his $K=\frac{1}{2}$ and $K=\frac{3}{2}$ bands are considered to arise from the same intrinsic state in the present analysis. The major difference between the two analyses is the wave function of the $I=\frac{1}{2}$ ground state.

In Table I, we give the values of $\langle a+b\rangle$ and $\langle B\rangle$ computed from the energy level spacings and the measured value of the ground-state magnetic moment for W^{183}. We compare this[1] with the values of the equivalent quantities for $\delta=0.2$, $\epsilon=0$ and with the values obtained for $\delta=0.215$, $\epsilon=0.026$. From Table I,

[6] A. K. Kerman, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. **30**, No. 15 (1956).

TABLE I. Properties of single-particle wave function.

	Semiempirical or measured	Calculated[a] for $\delta=0.2$, $\epsilon=0$	Calculated for $\delta=0.215$, $\epsilon=0.026$
$\langle a+b\rangle$[b]	+0.16	−0.20	−0.06
$\langle B\rangle$[c]	+1.25	+0.9	+0.9
μ (nm)	+0.12[d]	+0.8	+0.10

[a] See reference 1.
[b] Decoupling parameter.
[c] Coriolis interaction matrix element.
[d] P. B. Sogo and C. D. Jeffries, Phys. Rev. 98, 1316 (1955).

it is clear that the nonzero value of ϵ has improved the value of $\langle a+b\rangle$ relative to the axially symmetric value while removing the very large discrepancy between observed and calculated magnetic moments. We have computed the values of these matrix elements for several other choices of δ and find, for $\mu\simeq 0.1$, $\langle a+b\rangle$ varies between -0.03 and -0.08, $\langle B\rangle$ lies between 0.8 and 1.0, and ϵ goes from 0.03 to 0.02 over the interval $0.15<\delta<0.30$. A value of δ near 0.2 seems most reasonable as a single-particle level which is mostly $\Omega=-\frac{7}{2}$ crosses our single-particle level in this region.

The hydrodynamic model[3] gives a relation between the quantities ω_M used in H_{sp} and the inertial constants A_M. For

$$\omega_M\propto[1+\beta\cos(\gamma-\tfrac{2}{3}\pi M)]^{-1}, \tag{23}$$

we have

$$A_M\propto[\sin^2(\gamma-\tfrac{2}{3}\pi M)]^{-1}, \tag{24}$$

where M takes on the values 1, 2, and 3. Because we chose to use positive values of ϵ, ω_1 and ω_2 are interchanged in our calculation as are A_1 and A_2. Equivalently, we use negative values of γ. In terms of δ and ϵ, we have

$$-\gamma\simeq 30°(3\epsilon/\delta). \tag{25}$$

For the values $\delta=0.215$ and $\epsilon=0.026$, which we used to fit the magnetic moment, we obtain $\gamma\simeq-11°$. However, from the analysis of the energy levels in W^{183}, we obtain $A^+=29.856$ keV and $A^-=0.324$ keV which, using Eq. (24), implies $\gamma\simeq-1°$. From our analysis of the energy levels, we cannot obtain a very reliable value of A_3. We get $\rho=184.55$ keV but $[\langle-3/2_R\rangle^2-\langle1/2_R\rangle^2]$ may be close to zero (it varies from about zero to 0.8 over the trajectory in δ, ϵ space which we have considered). The fact that we cannot get a reliable estimate of A_3 is unfortunate as we do not then have a real test of Eq. (24), i.e., is any value of γ consistent with all three inertial constants.

In any event, there is a serious discrepancy between the value of γ implied by our single-particle potential and that obtained from the analysis of the energy levels. We cannot argue that $\gamma=0$, A_1 and A_2 are the inertial constants for rotational bands based on two different single-particle states, as does Kerman,[6] because this leads to such a large error in the calculation of the ground-state magnetic moment. We may argue that the Nilsson type of potential which we have used is not quantitative, as is indicated by the lack of complete agreement which we have between the semiempirical and calculated values of $\langle a+b\rangle$ and $\langle B\rangle$. Alternatively, we may argue that the relations between ω_M and A_M given in (23) and (24) are incorrect. There is evidence[7] from even-even nuclei casting doubt on the ability of (24) to give ratios of the inertial constants correctly.

We may argue that the discrepancy between the two values of γ arises from inaccuracies in our single-particle potential [e.g., the values of C and D in Eq. (2)]. The asymmetric oscillator wave function which gives the correct ground-state magnetic moment is mostly a mixture of the Nilsson levels $[510]\frac{1}{2}-$ and $[512]\frac{3}{2}-$, which are very close in energy for $\gamma=0°$. If the single-particle potential were modified in such a way as to bring an $\Omega=\frac{5}{2}$ level near these two levels, for $\gamma=0°$, it is quite conceivable that a small value of γ would lead to a single-particle wave function having the correct magnetic moment. We emphasize again that $\gamma=0°$ will not give the correct wave function, as $\gamma=0°$ gives states which are pure in Ω and the correct value of the magnetic moment depends crucially on the mixture of Ω states. It is also clear that the Coriolis interaction applied to symmetric oscillator wave functions will not give mixtures of Ω to a level which has $I=\frac{1}{2}$. Our problem is the magnitude of γ, but it is clearly nonzero.

Continuing with the assumption that the single-particle potential is inexact, we calculate the electromagnetic transition probabilities. Our assumption is that the single-particle wave function which we obtained for $\delta=0.215$ and $\epsilon=0.026$ is approximately correct and can be used to calculate all unknown matrix elements. Whenever possible, we use the values of matrix elements obtained from the energy levels as these are the values which we would presumably obtain if our potential were exact. Beside single-particle matrix elements, the values of γ and Q_0 enter into the calculation of the transition probabilities. We shall treat Q_0 as a free parameter; for γ, we insert the values $-1°$ and $-11°$ and calculate the transition probabilities for both cases.

Formulas for the reduced transition probabilities have been given[5] for the asymmetric rotor model, and we use the following set of numerical values for the single-particle matrix elements:

$$\begin{aligned}&\langle\Omega\rangle=-0.48, && \langle S^0\rangle=0.325,\\ &\langle S^-\rangle=-0.187, && \langle S^+\rangle=-0.144,\\ &\langle a+b\rangle=0.17, && \langle B\rangle=1.25.\end{aligned} \tag{26}$$

The first four numerical values are computed from our single-particle wave function, and the last two are taken from the analysis of the energy levels. For Q_0, we use the value 7×10^{-24} cm². The matrix element $\langle S^+\rangle$ is

[7] C. A. Mallman, Nucl. Phys. 24, 535 (1961).

TABLE II. Transition intensities.[a]

	Expt.[a]	Symmetric model[b]	$\gamma=-11°$	$\gamma=-1°$
$C\rightarrow A$	0.93	1.4	0.98	1.05
$C\rightarrow B$	1.0	1.0	1.0	1.0
$D\rightarrow B$	0.44	0.36	0.54	0.51
$D\rightarrow C$	1.0	1.0	1.0	1.0
$E\rightarrow A$	0.17	0.45	0.32	0.46
$E\rightarrow B$	1.0	1.0	1.0	1.0
$E\rightarrow C$	0.10	0.17	0.25	0.26
$F\rightarrow A$	1.0	1.0	1.0	1.0
$F\rightarrow C$	0.045	0.10	0.14	0.28
$F\rightarrow D$	0.22	0.36	0.24	0.38
$F\rightarrow E$	0.035	0.10	0.19	0.31
$G\rightarrow C$	1.0	1.0	1.0	1.0
$G\rightarrow D$	0.036	0.036	0.07	0.06
$H\rightarrow B$	0.12	0.21	0.09	0.18
$H\rightarrow C$	1.0	1.0	1.0	1.0
$H\rightarrow D$	0.12	0.16	0.13	0.16
$H\rightarrow E$	0.031	0.07	0.09	0.10
$H\rightarrow F$	0.01	0.01	0.05	0.06
$H\rightarrow G$	0.007	0.02	0.03	0.04

[a] We compute the relative intensities of all transitions from the same initial state. The most intense transition from each state is normalized to 1.0. Each transition is designated by the letters associated with the initial and final states in Fig. 1.
[b] See reference 6.

given[5] as

$$\langle S^{+}\rangle=\sum_{l,\Omega}{}'(-1)^{l}(a_{l,\Omega+\frac{1}{2},-\frac{1}{2}})(a_{l,-\Omega-\frac{1}{2},-\frac{1}{2}}). \quad (27)$$

In Table II, we compare the transition intensities for $\gamma=-1°$ and $\gamma=-11°$ with the experimental values and with those calculated by Kerman[6] for the axially symmetric model. It should be emphasized that the calculation of Kerman treats all of the single-particle matrix elements as adjustable, whereas the present treatment does not adjust any of them to fit the observed transition intensities. From Table II, we see that the over-all agreement with experiment for $\gamma=-11°$ is roughly the same as that obtained by Kerman and for $\gamma=-1°$, the agreement is not quite as good. This suggests that the value of γ as given by the wave function may be more accurate than that obtained from the moments of inertia.

The ground-state spin of Os^{187} is $\frac{1}{2}$, and its measured magnetic moment is about the same as that of W^{183}. We may account for this magnetic moment by assuming that both nuclei have the same ground-state wave functions.

Another spin $\frac{1}{2}$ ground state with an anomalously low magnetic moment occurs[2] in Fe^{57}. When we examine a Nilsson diagram for the $\Omega=\frac{1}{2}$ single-particle state which should be occupied by the last neutron in Fe^{57}, we find a nearby $\Omega=\frac{3}{2}$ state. The measured magnetic moment[8] of the ground state is 0.09 nuclear magneton, and we find that for $\epsilon/\delta\simeq0.2$ ($\gamma\simeq-20°$) we can account for this value quite satisfactorily for $0.2<\delta<0.3$. $\gamma\simeq-20°$ is consistent with the value[9] reported for Fe^{56}.

The fact that an asymmetric harmonic oscillator potential removes the discrepancies between the theoretical and experimental magnetic moments for W^{183}, Os^{187}, and Fe^{57} is strong evidence for the inclusion of the asymmetric term, ϵ, in the nuclear Hamiltonian.

[8] G. W. Ludwig and H. H. Woodbury, Phys. Rev. **117**, 1286 (1960).
[9] E. D. Klema, C. A. Mallman, and P. P. Day, Nucl. Phys. **25**, 266 (1961).